Exercises for Dynamics I, 11. January 2007

1. Consider a flow with concentric, circular streamlines.
What velocity pattern is necessary if the vorticity is to vanish?
Is the same result valid for cyclones and anticyclones?

2. Evaluate the absolute vorticity $\zeta_a = \zeta + f$ for typical values in mid-latitude flow to show that generally $f > |\zeta|$. For the Northern Hemisphere: find numerical values that would make the absolute vorticity negative.

3. Stationary Rossby waves: Consider the vorticity equation

$$\frac{D}{Dt}(\zeta + f)/h = 0 \tag{1}$$

with h =const., u and v are the velocity components. Assume that

$$u = U = const > 0 \tag{2}$$
$$v = v(x,t) \tag{3}$$

Linearize this equation around a basic state and derive the vorticity equation!

With the ansatz

$$v(x,t) = A\cos[\frac{2\pi}{L}(x - ct)] \tag{4}$$

determine whether any combination of c and L is possible.

What is the wavelength L_S of the stationary wave?

4. Stability: Consider a dynamical system

$$\frac{d}{dt}x = Ax \tag{5}$$

with a 2x2 matrix A

$$a_{11} = -\lambda \tag{6}$$
$$a_{12} = 10 \tag{7}$$
$$a_{21} = 0 \tag{8}$$
$$a_{22} = \mu \tag{9}$$

Determine the conditions for stability!

Write down the Euler-scheme.

Solve the equation numerically for $\lambda = 2, \mu = -1$ with initial condition $x_1(0) = 10, x_2(0) = \pm 2$.