

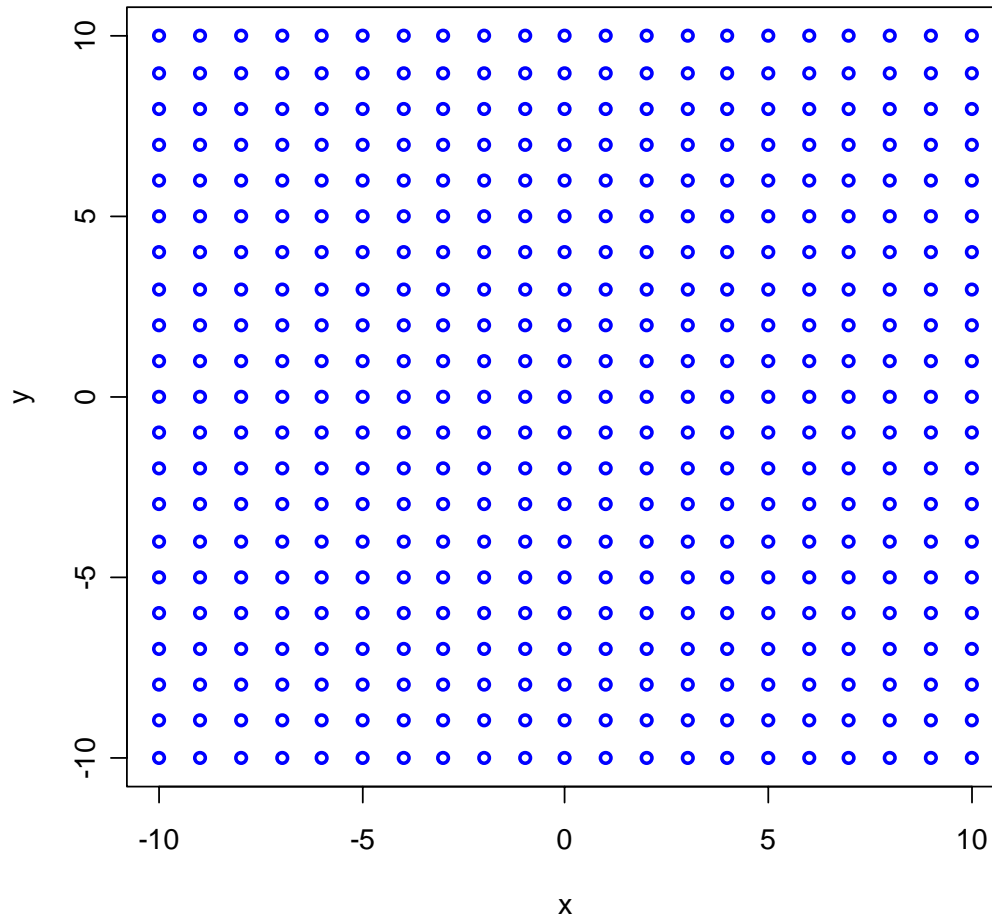
A very short introduction into the numerical solution of vector field problems

How to solve such a problem numerically

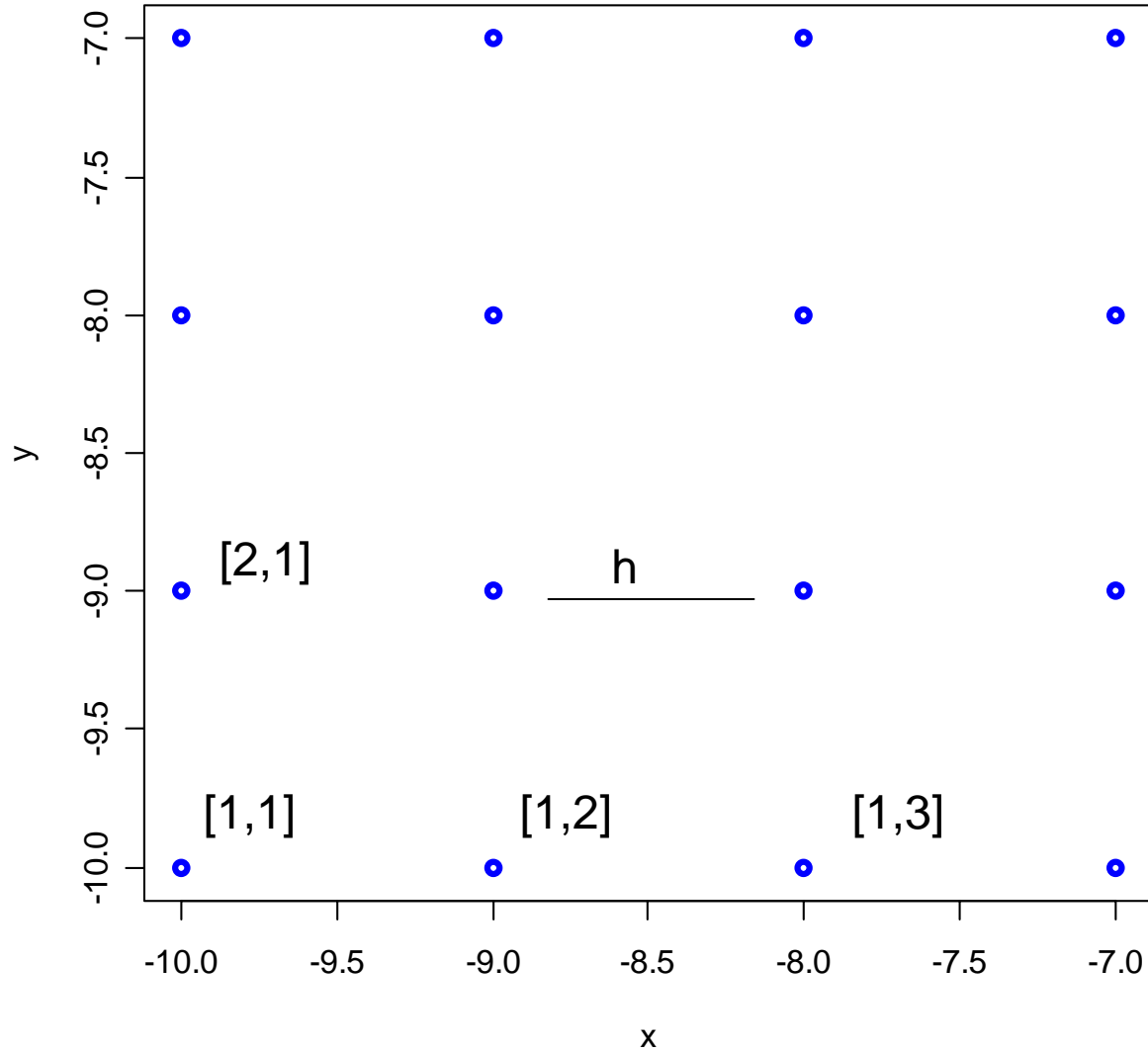
- work on a discrete grid
- replace differentials by differences
- replace integrals by sums

- Example of an R code

Work on a discrete grid

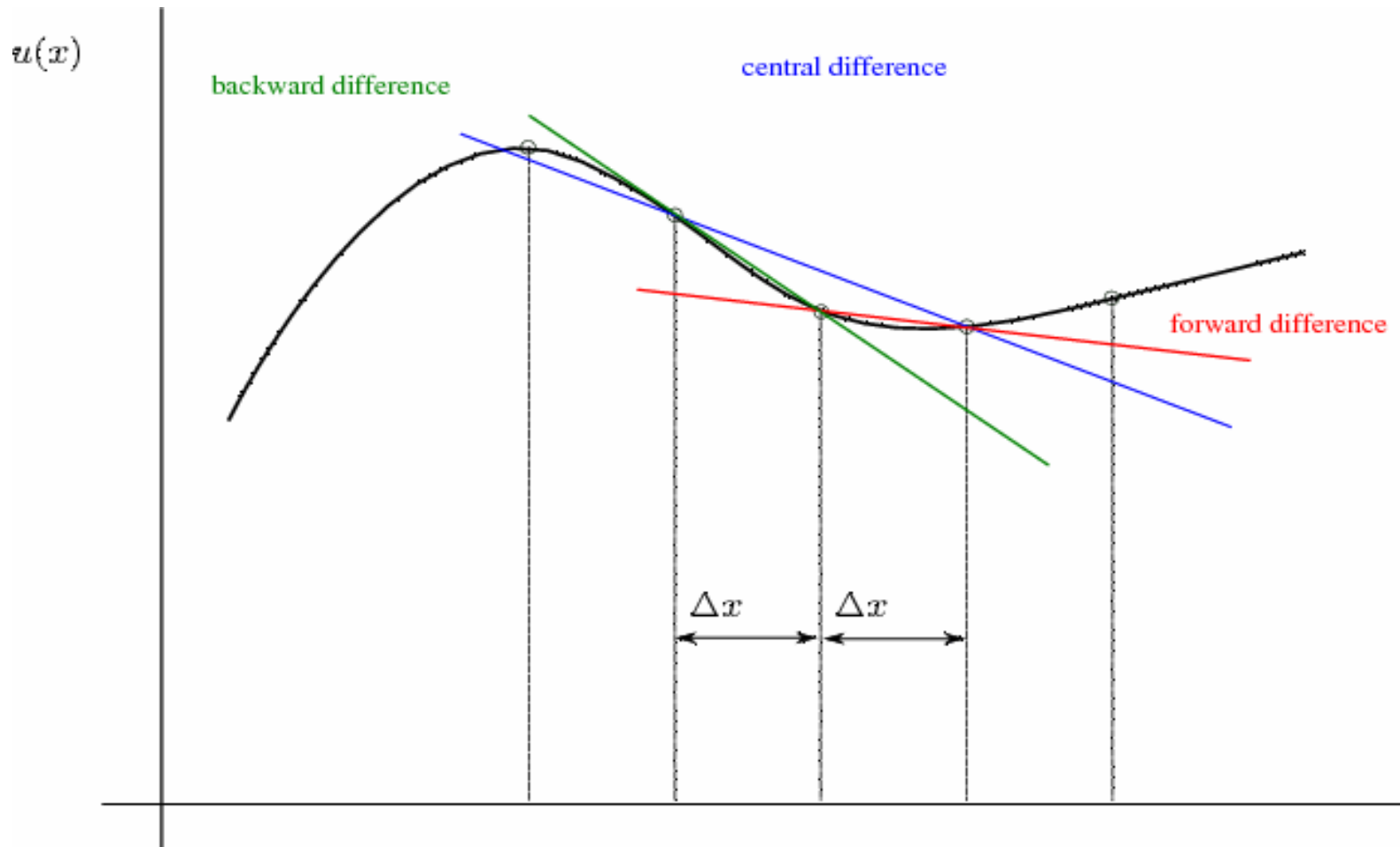


In this exercise we use a 21x21 grid, from $(-10,-10)$ to $(10,10)$



The data is stored in a 2D matrix , e.g. the index [1,1] accesses the value of the point at position -10,-10

Replace derivatives by differences

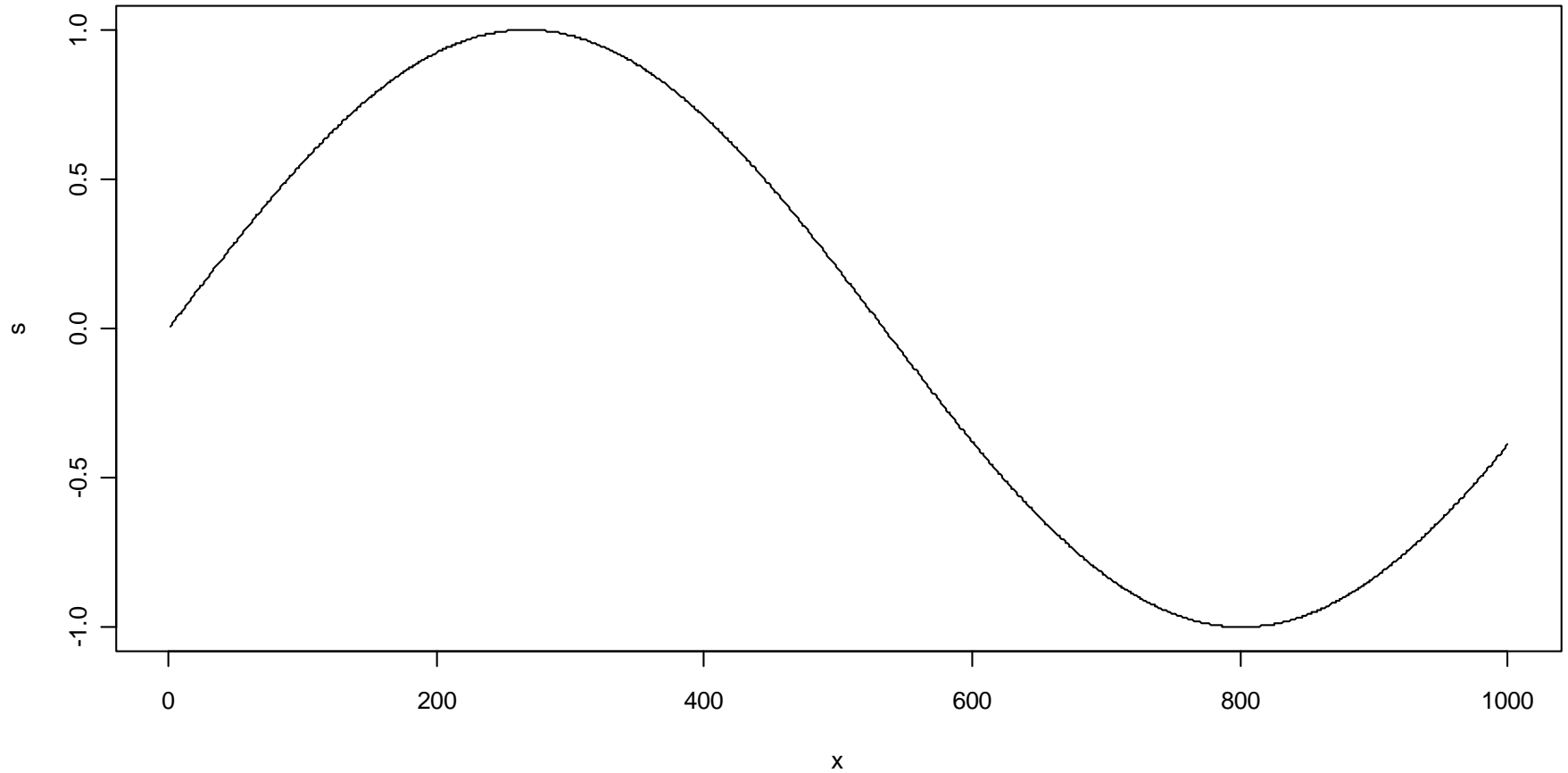


We use the central difference scheme in this exercise:

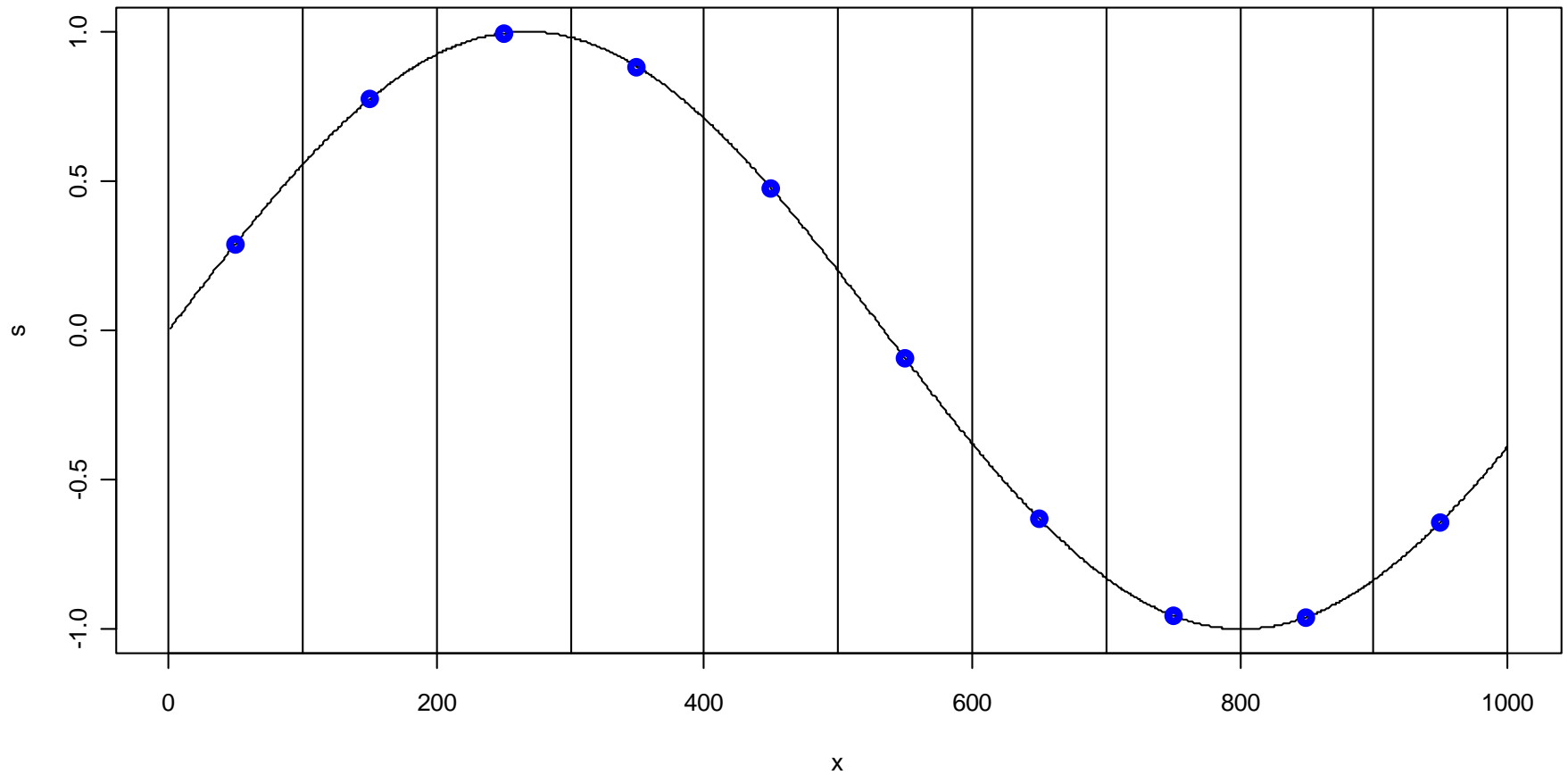
$$f_Z(x) = \frac{f(x+h) - f(x-h)}{2h}$$

zentraler DQ

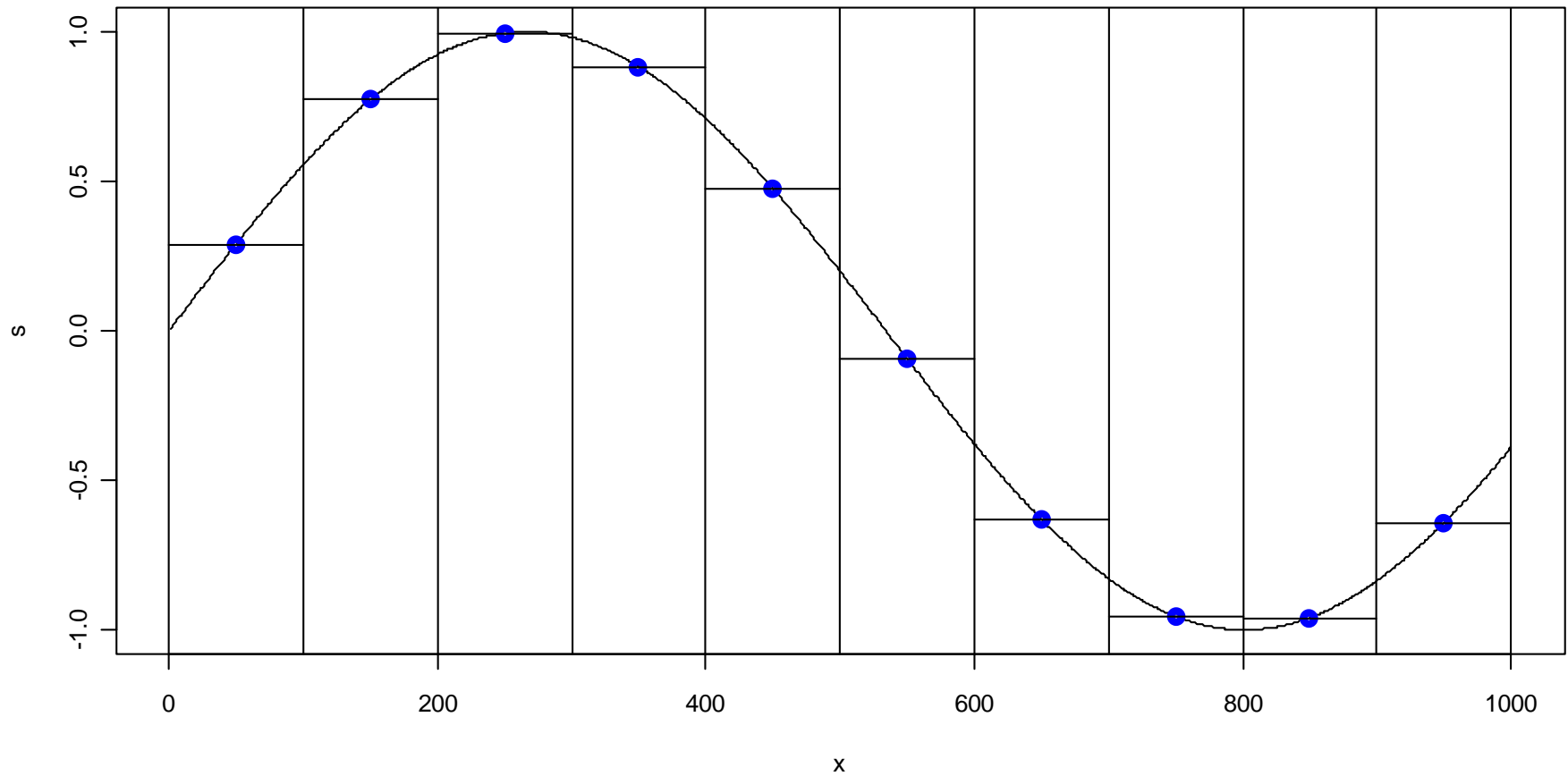
Replacing integrals by sums



Replacing integrals by sums



Replacing integrals by sums



Simplifications for this exercise

- Only 2D problems
- Fixed grid of 21x21 points
- $h = 1$

Definitions / variable names:

s = scalar field

v_x, v_y = two components of a vector field

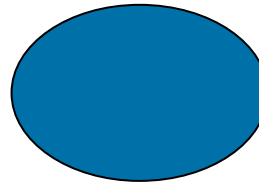
iX, iY = index of the discrete grid

x, y = midpoints of the gridboxes

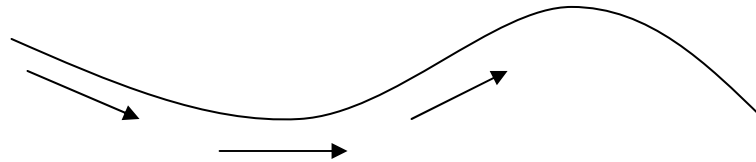
Provided functions

■ `diff.x`, `diff.y` : Differentiate the scalar field in x/y direction

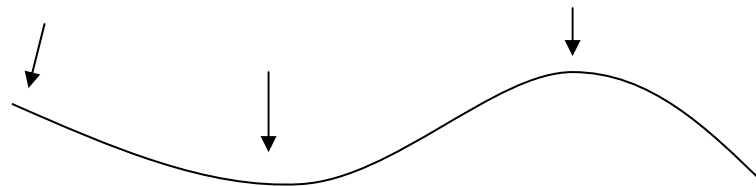
■ `VolumeIntegral`



■ `LineIntegral`



■ `AreaIntegral`



■ `quiver` = vector plots

example: source code of diff.x

```
■ #Differentiation of a scalar 2D field s[x,y] in x direction
■ #It assumes that the distance between the gridpoints in each direction is 1 (h=1)
■ diff.x<-function(s)
■ {
■     Nx<-dim(s)[1] #Size of the field in X direction
■     Ny<-dim(s)[2] #Size of the field in Y direction

■     h<-1      #h = 1

■     result<-matrix(NA,Nx,Ny) #Definiton a Nx x Ny field, filled with NA's = missing values

■     for (ix in 2:(Nx-1)) #loop through all cells
■         for (iy in 1:Ny)
■             {
■                 result[ix,iy]<-(s[ix+1,iy]-s[ix-1,iy])/(2*h) #symetric difference
■             }

■     return(result)
■ }
```

How to start

- Load `exercice1.R` into the R workspace
- Save the file using another name, to keep the original version when modifying the file
- Execute the whole file (CTRL-A to mark everything, CTRL-R to run it), than all functions are in the memory
- Get a rough overview of the code:
- Where does the main program start ?
- Where is the vector field initialized