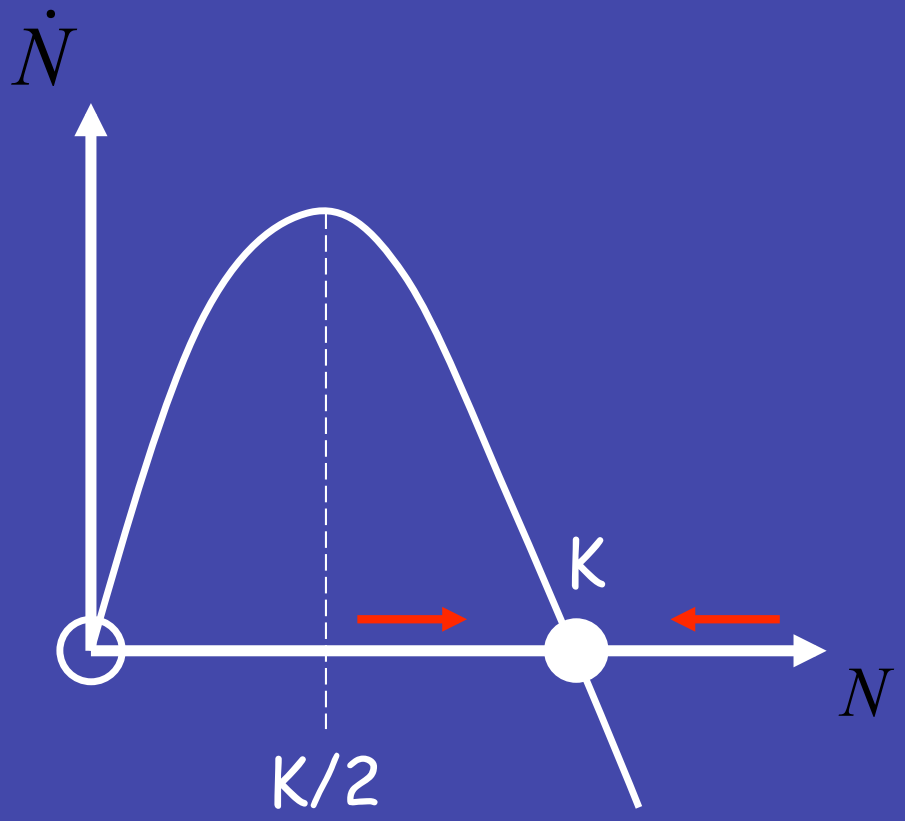
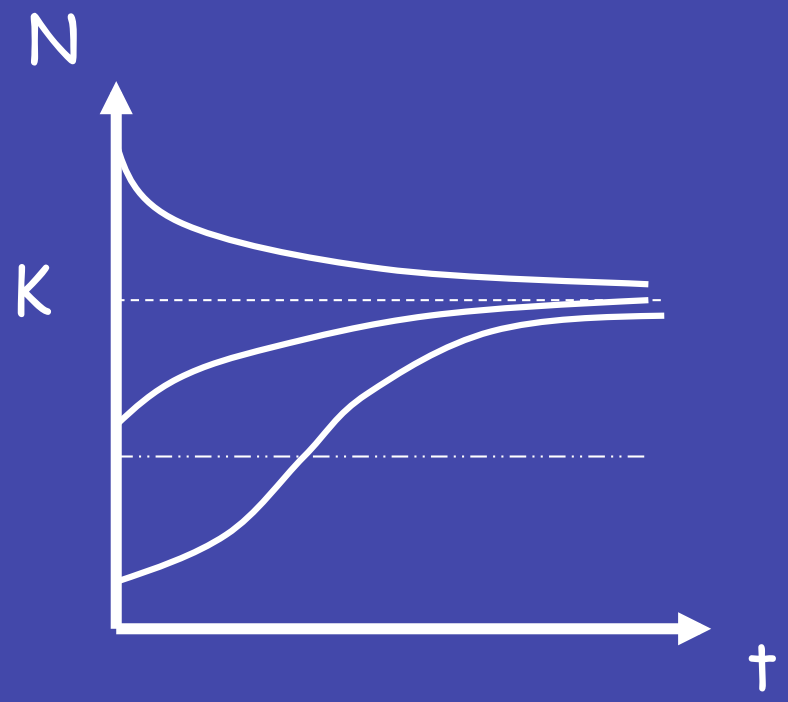
Logistic Differential Equation

$$\dot{N} = rN\left(1 - \frac{N}{K}\right)$$



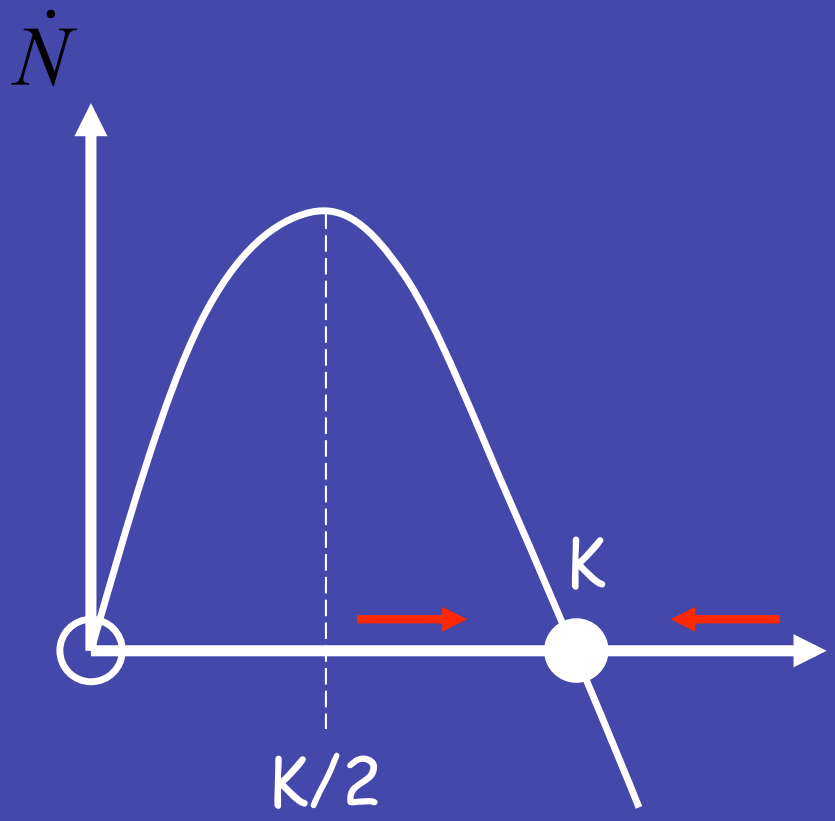


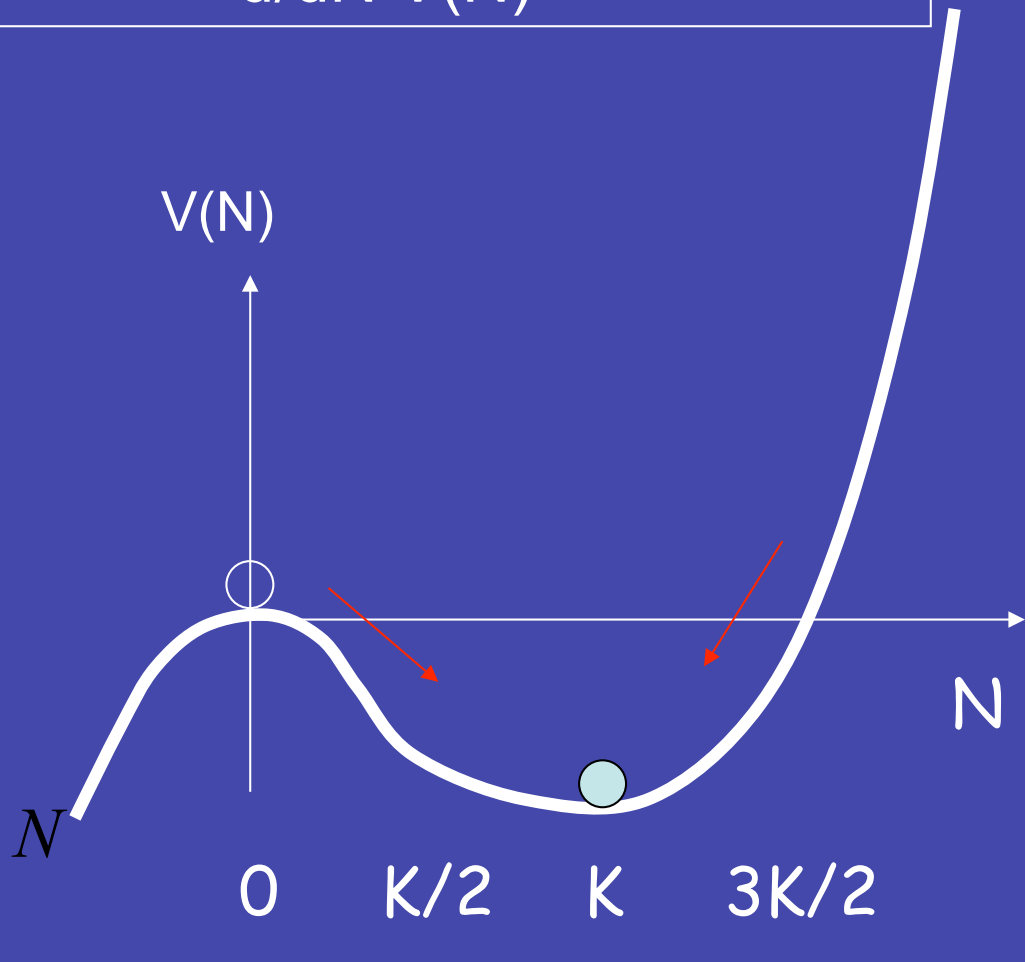
Logistic Differential Equation

$$\dot{N} = rN\left(1 - \frac{N}{K}\right)$$

d/dt N = - d/dN [r N^2 (N/3k -1/2)]

= - d/dN V(N)





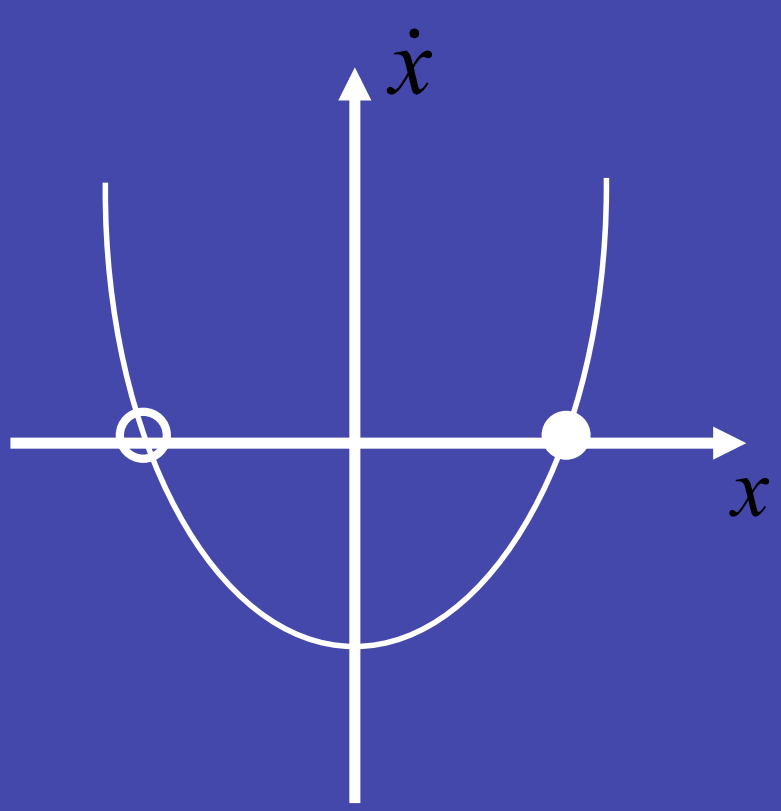
What is a bifurcation?

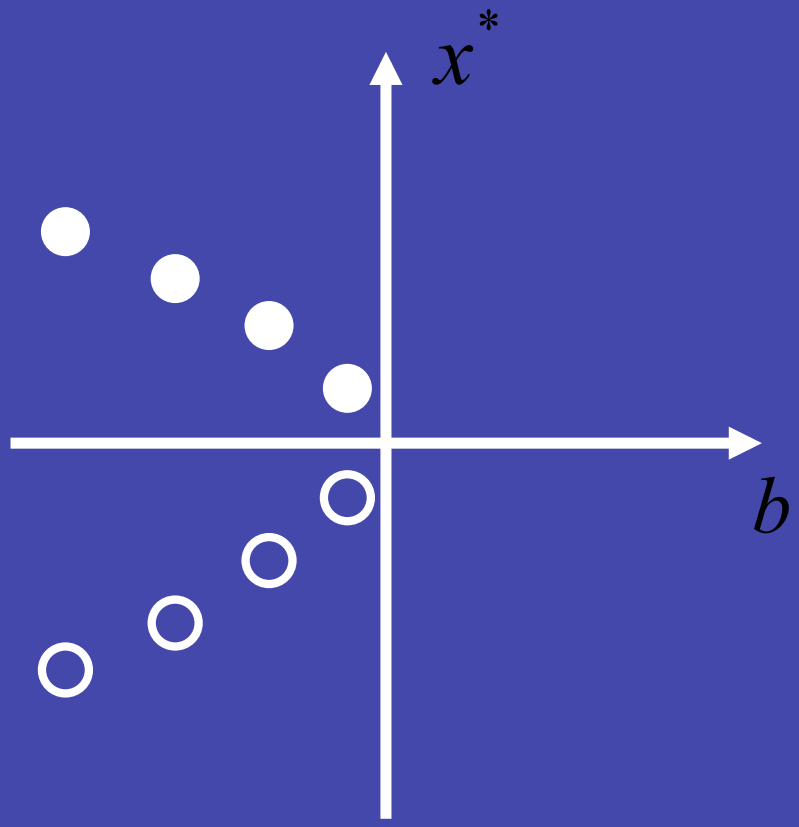
Max & Moritz,
W. Busch

A qualitative change in the solution !

Saddle Node Bifurcation (1-dim)

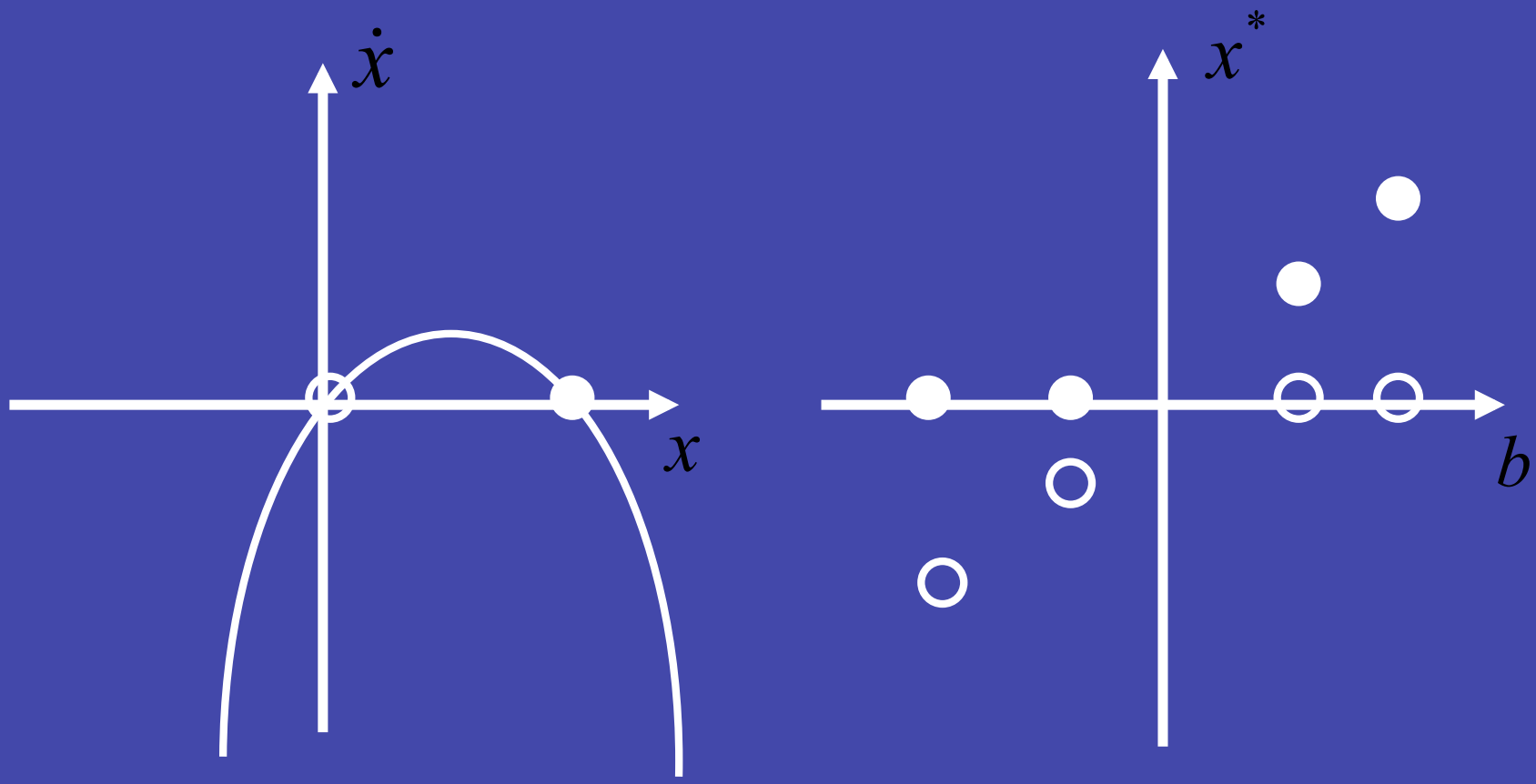
Prototypical example: $\dot{x} = b + x^2$





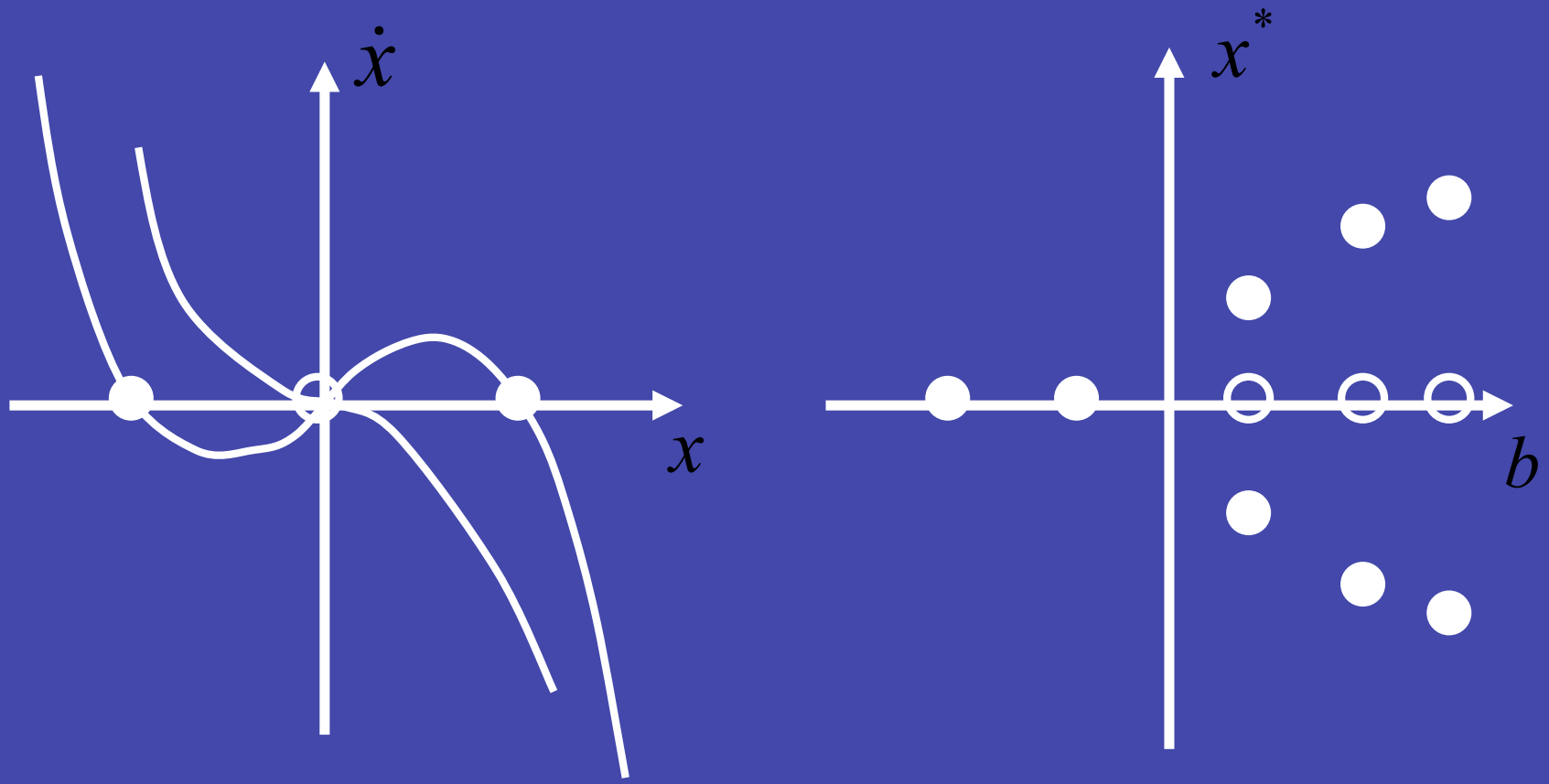
Transcritical Bifurcatoin

Prototypical example: $\dot{x} = bx - x^2$



Pitchfork Bifurcation

Prototypical example: $\dot{x} = bx - x^3$

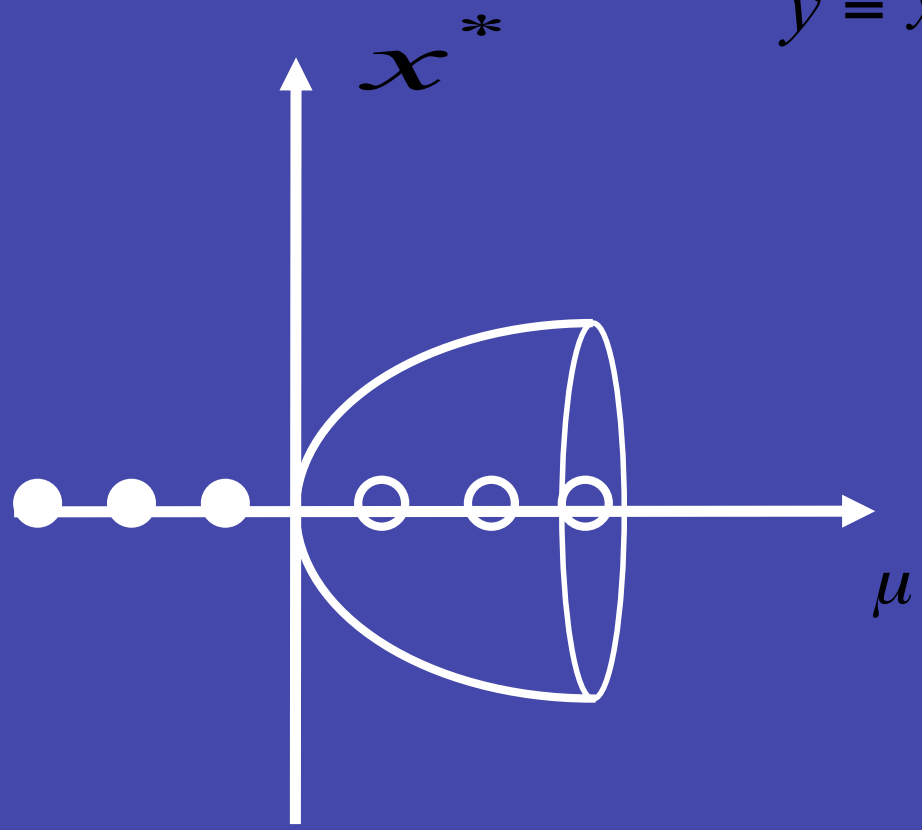


Hopf-Bifurcation

Prototypical example:

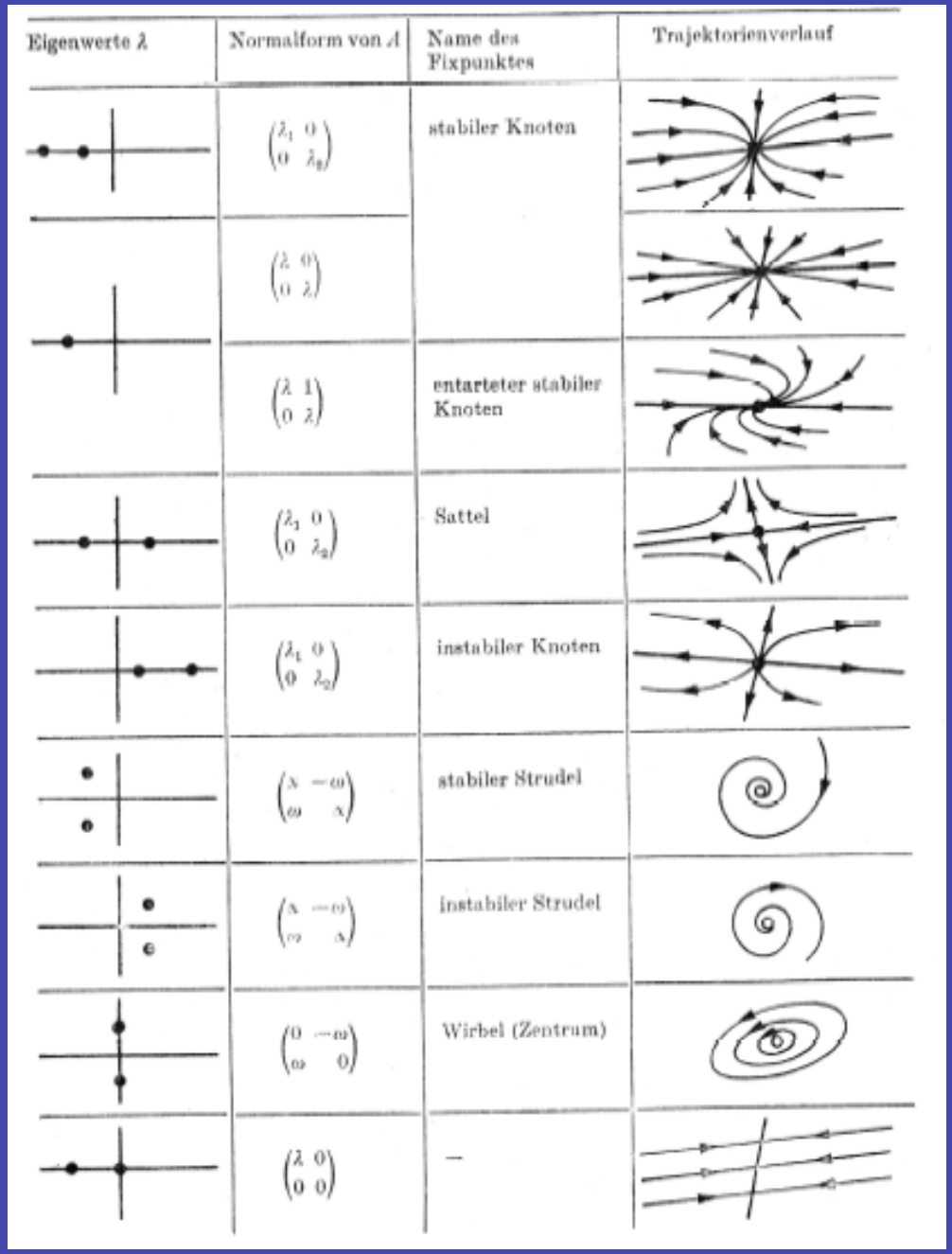
$$\dot{x} = \mu x - y - x(x^2 + y^2)$$

$$\dot{y} = x - \mu y - y(x^2 + y^2)$$



Dynamics of Two Dimensional Systems

1. Find the fixed points in the phase space!
2. Linearize the system about the fixed points!
3. Determine the eigenvalues of the Jacobian.



Eigenwerte λ	Normalform von A	Name des Fixpunktes	Trajektorienverlauf
	$\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}$	stabiler Knoten	
	$\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}$		
	$\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}$	entarteter stabiler Knoten	
	$\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}$	Sattel	
	$\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}$	instabiler Knoten	
	$\begin{pmatrix}\lambda & -\omega\\ \omega & \lambda\end{pmatrix}$	stabiler Strudel	
	$\begin{pmatrix}\lambda & -\omega\\ \omega & \lambda\end{pmatrix}$	instabiler Strudel	
	$\begin{pmatrix}0 & -\omega\\ \omega & 0\end{pmatrix}$	Wirbel (Zentrum)	
	$\begin{pmatrix}\lambda & 0\\ 0 & 0\end{pmatrix}$	–	