Dynamics II: Barotropic Ocean Dynamics, 4. June 2007

1) The Sverdrup and Ekman transports and Ekman pumping velocity: are given by

$$\beta V = \mathrm{curl}(\tau_0/\rho_0) = -\frac{\partial}{\partial y}\tau_0^x/\rho_0 \tag{1}$$

$$fV_E = -\tau_0^x/\rho_0 \tag{2}$$

$$w_E = \mathrm{curl}(\tau_0/f\rho_0) = -\frac{\partial}{\partial y}(\tau_0^x/f\rho_0) \tag{3}$$

where the windstress vector is taken zonal. Assume $\tau_0^x = -\tau_0 \cos \pi y/B$ for an ocean basin $0 < x < L, 0 < y < B$.

a) at what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4}$ s^{-1} and $\beta = 210^{-11}$ m^{-1}s^{-1} and $B = 4000$ km, $\tau_0/\rho_0 = 10^{-4}$ m^{2}s^{-2}.

b) calculate the maximum of w_E for constant f (value see above). Is this measurable?

2) The Stommel model: of the wind-driven circulation in a homogeneous ocean of constant depth h is described by

$$R\nabla^2\psi + \beta\partial_x\psi = \mathrm{curl}(\tau_0/\rho_0) \tag{4}$$

$$= \partial_x\tau_0^y/\rho_0 - \partial_y\tau_0^x/\rho_0 \tag{5}$$

where R is a coefficient of bottom friction, β the derivative of the Coriolis frequency at a central latitude, and the τ_0 the windstress vector. Finally, ψ is the streamfunction of the depth integrated velocity

$$U = (U, V) = \int_{-h}^{0} u dz$$

i.e.

$$U = -\partial_y\psi, V = \partial_x\psi$$

a) Derive this equation from the conservation of momentum (linearized) and mass (volume!) assuming $w = 0$ at the mean surface $z = 0$ and at the bottom $z = -h$. For simplicity take Cartesian coordinates for the horizontal, $\beta = df/dy$. Boundary condition for the flux of momentum are $\tau(z = 0) = \tau_0$ and $\tau(z = -h) = R(-V, U)$.

b) in the boundary layer the terms on the left hand side of (4) get large. Show by scaling that the width of the layer is $W = R/\beta$.

c) how large must R be to get a width $W = 100$ km? ($\beta = 2 \times 10^{-11}$ m^{-1}s^{-1}).

3) The Munk model: is obtained if horizontal friction is considered in (4) instead of bottom friction. The Munk model is

$$-A_h\nabla^4\psi + \beta\partial_x\psi = \mathrm{curl}(\tau_0/\rho_0) = \partial_x(\tau_0^y/\rho_0) - \partial_y(\tau_0^x/\rho_0) \tag{6}$$

where $\nabla^4 = (\nabla^2)^2$ denotes $\partial^4/\partial x^4 + 2\partial^4/(\partial x^2\partial y^2) + \partial^4/\partial y^4$ and A_h is a coefficient describing horizontal diffusion of momentum. Derive (by a rough scale analysis of the left hand side) the width W of the western boundary layer for this model. Determine A_h from a width $W = 100$ km and $\beta = 2 \times 10^{-11}$ m^{-1}s^{-1}.