Dynamics II: Exam

Time: Monday, 23. July 2007, 10:00-12:00, 220 points, *100 % = 110 points*
Devices: one (!) sheet of paper which has to be delivered at the end; calculator (if you want).

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1) Several questions about the course (*30 points*).

Q1: What is the driving force for the atmosphere-ocean system?

3 points

Q2: List different sub-systems of climate and their typical timescales!

3 points

Q3: Please write down the barotropic potential vorticity equation for large-scale motion!

3 points

Q4: What is the definition of correlation and covariance?
How is the Fourier transformation of the covariance called?

3 points

Q5: Describe the upscaling technique!

3 points

Q6: Explain the δ notation for stable oxygen isotopes!

3 points

Q7: Provide a list of stable isotopes in nature!
Why are they important?

3 points

Q8: Explain dispersive and non-dispersive waves!

3 points

Q9: Calculate the following operations for the function

$$f(x, y, z) = x^3 + 3x - 4xz + z^4 \quad : \tag{1}$$

∇f, calculate the divergence of the result!

3 points

Q10: Calculate the rotation of ∇f!

3 points

2) Energy

Start with the shallow water equations

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} \tag{2}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} \tag{3}$$

$$\frac{\partial \eta}{\partial t} + H(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0 \tag{4}$$

with H=const. as mean depth and η as surface anomaly.

a) Show that the total energy throughout the shallow water column satisfies

$$\frac{\partial(PE+KE)}{\partial t} + \nabla \cdot F = 0, \tag{5}$$

where

$$KE = \frac{1}{2}\rho H(u^2+v^2), \quad PE = \frac{1}{2}\rho g\eta^2, \text{and vector } F = \rho gH \quad (u\eta, v\eta). \tag{6}$$

b) Determine the dispersion relation for plane wave solutions to this problem of the form

$$(u_0, v_0, \eta_0)\exp i(kx+ly-\omega t) \tag{7}$$

and express u_0, v_0 in terms of η_0.

c) Let

$$\exp i(kx+ly-\omega t) = e^{i\Psi} \tag{8}$$

and let an overbar denote the average of the quantity over a full phase of the wave, i.e., over the interval $0 \le \Psi \le 2\pi$. Show that

$$\frac{\bar{K}E}{\bar{P}E} = \frac{\omega^2+f^2}{\omega^2-f^2}. \tag{9}$$

Under what circumstances is there an equipartion of energy $\bar{K}E = \bar{P}E$?

30 points

3) Rossby wave formula (long waves in the westerlies)

Consider the vorticity equation

$$\frac{D}{Dt}[(\zeta + f)/h] = 0 \tag{10}$$

with h =const., u and v are the velocity components.

a) Assume a mean flow with constant zonal velocity U

$$u = U = const > 0 \tag{11}$$

and a varying north-south component

$$v = v(x, t) \tag{12}$$

which gives the total motion a wave-like form. Derive the vorticity equation!

b) With the ansatz

$$v(x, t) = A\cos[(kx - \omega t)] \tag{13}$$

determine the disperion relation $\omega(k)$, group velocity $\frac{\partial\omega}{\partial k}$, and the phase velocity $c = \omega/k$.

c) Derive the wavelength $L = 2\pi/k$ of the stationary wave given by $c = 0$.

d) A typical wavelength is 6000 km, a typical U is 15 m/s. Does the wave propagate from east to west or opposite?

30 points

4) Barotropic wave:

A jet stream of speed 50m/s meanders with 6000 km wavelength and 1500 km amplitude, centred at 45· Compute the path (the meridional displacement) of the barotropic wave and find the phase speed.

The path can be approximated:

$$y = A\ \cos[2\pi(x - ct)/\lambda] \tag{14}$$

$$2\Omega/R_{earth} = 2.29 \cdot 10^{-11} m/s \tag{15}$$

$$c_0 = -\beta(\frac{\lambda}{2\pi})^2 \tag{16}$$

Write:

$\beta =?$

$c =?$

$y(x) =?$.

30 points

5) Ice Dynamics:

a) The forces, acting on a block of ice are given in terms of stresses. What are the two stress components and how is the stress tensor defined.

b) In glaciology the deviatoric stress is responsible for ice deformation. Give its definition.

c) Ice reacts like on external stresses by internal deformation. How is the general flow law after Glen defined.

d) Consider a column of ice with height H and unit cross-section perpendicular to an inclined plane of angle alpha. The weight of the ice column has a component parallel to the plane which is compensated by the basal shear stress. In general, ice becomes afloat at basal shear of 100 kPa. Estimate the thickness of a perfect plastic glacier, which surface slope is measured to be 10 deg? Use: $\rho = 917 kg/m^3, g = 9.81 m/s$

Answers:

a) normal stress σ_{ii} and shear stress $\tau_{ij}, i, j = x, y, z$

$$\sigma = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

b)

$$\tau_{ij} = \sigma_{ii} - \delta_{ij} 1/3(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

c)

$$\dot{\epsilon_{xy}} = A\tau^{n-1}\tau_{xy}, \qquad n = \text{flow-parameter} = 3$$

d)

$$\tau_b = \rho g H sin(\alpha)$$

$$\tau_b = 100 kPa, H = 64m$$

30 points

6) Ocean thermohaline circulation: a) Consider a geostrophic flow (u, v)

$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \tag{18}$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \tag{19}$$

with pressure $p(x, y, z, t)$.
Use the hydrostatic approximation

$$\frac{\partial p}{\partial z} = -g\rho \tag{20}$$

and equation (18) in order to derive the meridional overturning stream function $\Phi(y, z)$ as a fuction of density ρ at the basin boundaries! Φ is defined via

$$\Phi(y, z) = \int_0^z \frac{\partial \Phi}{\partial \tilde{z}}\, d\tilde{z} \tag{21}$$

$$\frac{\partial \Phi}{\partial \tilde{z}} = \int_{x_e}^{x_w} v(x, y, \tilde{z})\, dx \quad \text{(zonally integrated transport),} \tag{22}$$

where x_e and x_w are the eastward and westward boundaries in the ocean basin (think e.g. of the Atlantic Ocean). Units of Φ are $m^3 s^{-1}$. At the surface $\Phi(y, 0) = 0$.

b) Draw a figure of the Atlantic overturning!

10 points

7) Methods:

a) Consider the differential equation

$$\frac{d}{dt}x = ax - \epsilon x^2 \tag{23}$$

Explain the asymptotic method in case of $\epsilon << 0$. Expand x variables in a power series.

b) Provide an example from atmosphere-ocean dynamics!

10 points

8) Cells:

What are the names of the 3 meridional cells in the atmosphere? Draw a picture!

How is the equatorial cell driven? Are these cells geostrophically driven or not?

10 points

9) The Sverdrup and Ekman transports and Ekman pumping velocity: are given by

$$\beta V = \mathrm{curl}(\tau_0/\rho_0) = -\frac{\partial}{\partial y}\tau_0^x/\rho_0 \quad (24)$$

$$f V_E = -\tau_0^x/\rho_0 \quad (25)$$

$$w_E = \mathrm{curl}(\tau_0/f\rho_0) = -\frac{\partial}{\partial y}(\tau_0^x/f\rho_0) \quad (26)$$

where the windstress vector is taken zonal. Assume $\tau_0^x = -\tau_0 \cos \pi y/B$ for an ocean basin $0 < x < L, 0 < y < B$.

a) at what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4}$ s^{-1} and $\beta = 1.8 \cdot 10^{-11}$ m^{-1}s^{-1} and $B = 5000$ km, $\tau_0/\rho_0 = 10^{-4}$ m^{2}s^{-2}.

b) calculate the maximum of w_E for constant f (value see above). Is this measurable?

20 points

10) The Stommel model: of the wind-driven circulation in a homogeneous ocean of constant depth h is described by

$$R\nabla^2\psi + \beta\partial_x\psi = \mathrm{curl}(\tau_0/\rho_0) \quad (27)$$
$$= \partial_x\tau_0^y/\rho_0 - \partial_y\tau_0^x/\rho_0 \quad (28)$$

where R is a coefficient of bottom friction, β the derivative of the Coriolis frequency at a central latitude, and the τ_0 the windstress vector. Finally, ψ is the streamfunction of the depth integrated velocity

$$U = (U, V) = \int_{-h}^{0} u dz$$

i.e.

$$U = -\partial_y\psi, V = \partial_x\psi$$

a) Derive this equation from the conservation of momentum (linearized) and mass (volume!) assuming $w = 0$ at the mean surface $z = 0$ and at the bottom $z = -h$. For simplicity take Cartesian coordinates for the horizontal, $\beta = df/dy$. Boundary condition for the flux of momentum are $\tau(z = 0) = \tau_0$ and $\tau(z = -h) = R(-V, U)$.

b) in the boundary layer the terms on the left hand side of (27) get large. Show by scaling that the width of the layer is $W = R/\beta$.

c) how large must R be to get a width $W = 100$ km? ($\beta = 2 \times 10^{-11}$ m^{-1}s^{-1}).

d) can you imagine a planet form or rotation conditions where the other circulations are possible for the Stommel gyre?

20 points