Dynamics II: Wind-driven Circulation, Cyclostrophic Wind

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1. Ekman layer: Consider a geostrophic flow $(\bar{u}, \bar{v})$

$$-f\bar{v} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x} \tag{1}$$

$$f\bar{u} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial y} \tag{2}$$

with pressure $\bar{p}(x, y, t)$. The boundary-layer equations are then

$$-f(v - \bar{v}) = \nu \frac{\partial^2 u}{\partial z^2} \tag{3}$$

$$f(u - \bar{u}) = \nu \frac{\partial^2 v}{\partial z^2} \quad . \tag{4}$$

The boundary conditions are specified to be at the surface

$$\rho_0 \nu \frac{\partial u}{\partial z} = \tau^x \tag{5}$$

$$\rho_0 \nu \frac{\partial v}{\partial z} = \tau^y \tag{6}$$

and for $z \to -\infty$: $u = \bar{u}, \quad v = \bar{v}$.

a) Calculate the flow (u,v) as the departure from the interior flow $(\bar{u}, \bar{v})$!

b) Calculate the net wind-driven horizontal transport through integration

$$V = \int_{-\infty}^{0} dz(v - \bar{v}) \tag{7}$$

$$U = \int_{-\infty}^{0} dz(u - \bar{u}) \quad . \tag{8}$$

What is the direction of U and V in terms of the surface wind stress τ?

c) For the case $f = f_0$ of constant Coriolis parameter, determine the divergence of the flow

$$\int_{-\infty}^{0} dz\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{9}$$

which is identical to the vertical velocity across the Ekman layer (since w(0)=0).

2. Cyclostrophic wind: When the flow is sufficiently near the equator so that f is small or when the Corolis force is negligible compared to the centripedal acceleration, the gradient wind equation becomes

$$\frac{v\mathbf{k} \times \mathbf{v}}{R} = -\frac{1}{\rho}\nabla_z p \tag{10}$$

where $\mathbf{k}$ is the unit vector in z direction, $\mathbf{v}$ is the velocity vector, v ist the meridional velocity, R Earth radius, ∇_z horizontal nabla operator.

a) Derive this equation!

b) What is the associated gradient wind equation including the Coriolis force?

c) What is the Rossby number?