Exercises

Baroclinic and barotropic instability

1. Solve for $\Psi_3^{'}$ *and* $\omega_2^{'}$ in terms of $\Psi_1^{'}$ for a baroclinic Rossby wave whise phase speed satisfies $c_2 = U_m - \dfrac{\beta}{\left(k^2 + 2\lambda^2\right)}$.

2. For the case $U_1 = -U_3$ and $k^2 = \lambda^2$ solve for $\Psi_3^{'}$ *and* $\omega_2^{'}$ in terms of $\Psi_1^{'}$ for marginally stable waves (δ=0).

3. Obtain a formula for the Rossby wave phase speed in an homogenous barotropic fluid confined between two rigid horizontal lids when friction is included in the form **Fr=-μV,** where μ is a constant drag coefficient and **Fr** designates the horizontal friction force.

4. Suppose that a baroclinic fluid is confined between two rigid horizontal lids in a rotating tank in which β =0 but frictional drag must be included. If the frictional force has the same form as given in problem 3(that is, frictions is everywhere linearly proportional to the velocity) show that the two-level model perturbation vorticity equations in Cartesian coordinates can be written as:

$$\left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x} + \mu\right)\frac{\partial^2 \psi_1^{'}}{\partial x^2} - \frac{f}{\Delta p}\omega_2^{'} = 0$$

$$\left(\frac{\partial}{\partial t} + U_3 \frac{\partial}{\partial x} + \mu\right)\frac{\partial^2 \psi_3^{'}}{\partial x^2} + \frac{f}{\Delta p}\omega_2^{'} = 0$$

where perturbations are assumed in the form:

$$\psi_1 = -U_1 y + \psi_1^{'}(x,t)$$

$$\psi_3 = -U_3 y + \psi_3^{'}(x,t)$$

$$\omega_2 = \omega_2^{'}(x,t)$$

Assuming solutions of the form: $\psi_1^{'} = Ae^{ik(x-ct)}$, $\psi_3^{'} = Be^{ik(x-ct)}$, $\omega_2^{'} = Ce^{ik(x-ct)}$ show that the phase speed satisfies a relationship similar to $c = U_m - \dfrac{\beta\left(k^2 + \lambda^2\right)}{k^2\left(k^2 + 2\lambda^2\right)} \pm \delta^{1/2}$, with replaced everywhere by $i\mu k$ and that as a result the condition for baroclinic instability becomes:

$$U_T > \frac{\mu}{\left(2\lambda^2 - k^2\right)^{1/2}}$$