Course: Dynamics 1
Exercise: 9. November 2006

1. Examine the double vector product $\vec{\Omega} x(\Omega x \vec{r})$ with vectors $\Omega=(0,0,\omega)$, $r=(x,y,z)$.

2. Consider the heat diffusion-advection equation

$$\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial^2 x}+u\frac{\partial T}{\partial x}$$

and determine the time evolution with initial conditions

a) $T(x,0)=\exp(-x/a^2)$ with a=constant.
b) $T(x,0)=T_0$ for $x\geq 0$ and $T(x,0)=0$ elsewhere.
Discuss the special cases k=0 (no diffusion) and u=0 (no advection).

3. A tornado rotates with a constant angular velocity ω . Show that the surface pressure at the center of the tornado is given by:

$$p=p_0\exp(-\omega^2 r_0^2/(2\mathrm{RT}))$$

where p_0 is the surface pressure at the distance r_0 from the center and T is the temperature (assumed constant). [Hint: What are the dominant forces? Pressure gradient or centrifugal force.]
If the temperature is 288K, the pressure at 100m from the center is 100 kPa and the wind speed at 100m from the center is 100m/s, what is the central pressure?

4. Suppose a 1kg parcel of dry air is rising at a constant vertical velocity. If the parcel is being heated by radiation at a rate of $10^{-1} W/kg$, what must the speed of rise be in order to maintain the parcel at a constant temperature ? [Hint: Energy equation.]

5. Show that for an atmosphere with an adiabatic lapse rate (i.e. constant potential temperature), the geopotential $Z(z)=\Phi(z)/g_0$ is given by

$$Z=H_\theta[1-(p/p_0)^a]$$

where p_0 is the pressure at $Z=0$ and $H_\theta=c_p\theta/g_0$ is the total geopotential in the atmosphere $a=R/c_p$.