# Exercises; Dynamics I; 1. February 2007

1. The vorticity equation in Cartesian coordinates is:

$$\frac{d}{dt}(\zeta+f)=-(\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)+\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y}-\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

with $\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}$ . Using scale analysis show that for synoptic scale motions this equation can be approximated as : $\frac{d_h(\zeta+f)}{dt}=-f\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$ with $\frac{d_h}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}$ .

2. Show that the solenoidal term of the vorticity equation can be written as:

$$\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y}-\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)=-(\nabla \alpha \, x \, \nabla p)\vec{k}$$

3. For the equation

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\zeta'_g+\beta v'_g=\frac{-f_0}{H}\bar{u}\frac{\partial h_T}{\partial x}$$

consider solutions: $h_T(x,y)=\Re[h_0\exp(ikx)]\cos(ly)$ and $\psi(x,y)=\Re[\psi_0\exp(ikx)]\cos(ly)$ .
Show that amplitudes satisfy the relation:

$$\psi_0=\frac{f_0 h_0}{H(K^2-K_s^2)}$$

where $K^2=k^2+l^2$ and $K_s^2=\beta/\bar{u}$ .

4. Find the stationary solutions of the Lorenz system

$$\dot{x}=s(y-x)$$
$$\dot{y}=rx-y-xz$$
$$\dot{z}=xy-bz$$

How does the stability of the stationary solutions depend on the control parameters s, r and b?