

Dynamics I: Preparation and Exercises for the exam

Bremen/Bremerhaven, 8. Febr. 2007

Time: Monday, 26. Febr. 2007, 10:00-11:45, Room H1

Devices: one (!) sheet of paper which has to be delivered at the end; calculator (if you want).

Structure: 4 parts

1) Several questions about the course (ca. 20 min). **Some key words:** Coriolis force, mass budget, momentum equations, hydrostatic approximation, equation of state, friction, scale analysis, numerical scheme, advection and diffusion, vorticity dynamics, Ekman layer, thermohaline circulation, Kelvin waves, Rossby waves, gravity waves, topographic waves, stability, bifurcation analysis, potential vorticity, thermal wind, geostrophy, upwelling, wind driven ocean circulation.

2) One problem out of 1.-4. (25 min).

3),4) Probably two other problems (2 x 30 min).

1. Ocean thermohaline circulation: Consider a geostrophic flow (u, v)

$$-fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} \quad (1)$$

$$fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} \quad (2)$$

with pressure $p(x, y, z, t)$.

a) Use the hydrostatic approximation $\frac{\partial p}{\partial z}$ and equation (1) in order to derive the meridional overturning stream function $\Phi(y, z)$. Φ is defined via

$$\Phi(y, z) = \int_0^z \frac{\partial \Phi}{\partial \tilde{z}} d\tilde{z} \quad (3)$$

$$\frac{\partial \Phi}{\partial \tilde{z}} = \int_{x_e}^{x_w} v(x, y, \tilde{z}) dx \quad (\text{zonally integrated transport}), \quad (4)$$

where x_e and x_w are the eastward and westward boundaries in the ocean basin (think e.g. of the Atlantic Ocean). Units of Φ are $m^3 s^{-1}$. At the surface $\Phi(y, 0) = 0$.

b) Another definition of $\Phi(y, z)$ is via

$$-\frac{\partial \Phi}{\partial y} = \int_{x_e}^{x_w} w dx \quad (5)$$

with vertical velocity w . Why is the definition of a streamfunction useful? What is the physical law and equation behind that?

c) Consider now a water planet with flat bottom (unlike the Earth). Provide the meridional overturning stream function $\Phi(y, z)$ in this ocean! Is there a meridional transport due to geostrophy in the atmosphere?

2. Ekman layer: Consider a geostrophic flow (\bar{u}, \bar{v})

$$-f\bar{v} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} \quad (6)$$

$$f\bar{u} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial y} \quad (7)$$

with pressure $\bar{p}(x, y, t)$. The boundary-layer equations are then

$$-f(v - \bar{v}) = \nu \frac{\partial^2 u}{\partial z^2} \quad (8)$$

$$f(u - \bar{u}) = \nu \frac{\partial^2 v}{\partial z^2} \quad (9)$$

The boundary conditions are specified to be at the surface

$$\rho_0 \nu \frac{\partial u}{\partial z} = \tau^x \quad (10)$$

$$\rho_0 \nu \frac{\partial v}{\partial z} = \tau^y \quad (11)$$

and for $z \rightarrow -\infty$: $u = \bar{u}$, $v = \bar{v}$.

a) Calculate the flow (u, v) as the departure from the interior flow (\bar{u}, \bar{v}) !

b) Calculate the net wind-driven horizontal transport through integration

$$V = \int_{-\infty}^0 dz (v - \bar{v}) \quad (12)$$

$$U = \int_{-\infty}^0 dz (u - \bar{u}) \quad (13)$$

What is the direction of U and V in terms of the surface wind stress τ ?

c) For the case $f = f_0$ of constant Coriolis parameter, determine the divergence of the flow

$$\int_{-\infty}^0 dz \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \quad (14)$$

which is identical to the vertical velocity across the Ekman layer (since $w(0)=0$).

3. Bifurcation:

Consider two different versions of Stommel's box model of the ocean circulation:

$$\frac{d}{dt}x = a + bx|x - 1| \quad (15)$$

$$\frac{d}{dt}x = a + bx(x - 1)^2 \quad (16)$$

These equations are derived when the temperatures and surface fluxes are fixed. x denotes the normalized salinity difference between low and high latitudes (see lecture).

a) Calculate the bifurcation with respect to the parameters a, b for both (15) and (16)!

b) Draw the bifurcation diagram!

c) Consider the discretised form of equation (16) using the Euler scheme

$$\frac{d}{dt}x \approx \frac{x_{n+1} - x_n}{\Delta t} \quad (17)$$

d) Write down the iteration x_{n+1} as a function of x_n for equation (16)!

4. Rossby, gravity, and Kelvin waves:

Start with the shallow water equations

$$\frac{\partial u}{\partial t} - fv = -g \frac{\partial \eta}{\partial x} \quad (18)$$

$$\frac{\partial v}{\partial t} + fu = -g \frac{\partial \eta}{\partial y} \quad (19)$$

$$\frac{\partial \eta}{\partial t} + H \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (20)$$

with $H = \text{const.}$ as mean depth and η as surface anomaly.

a) with the assumption

$$\frac{\partial \eta}{\partial t} = 0$$

derive the dispersion relation for divergence-free Rossby waves! The restoring force is related to the Coriolis force. Ansatz: Introduce a streamfunction for u, v :

$$\Psi \sim \exp(ikx + ily - i\omega t)$$

b) with the assumption of $f = f_0 = 0$ derive the dispersion relation for gravity waves! The restoring force is related to the gravity force. Ansatz: take one of the equations (18,19,20) and derive the solution.

c) Inertia-Gravity wave (Poincare wave). As in b), but now $f = f_0 \neq 0$. Ansatz:

$$(u, v, \eta) = (\hat{u}, \hat{v}, \hat{\eta}) \times \exp(ikx + ily - i\omega t)$$

and derive algebraic equations for $(\hat{u}, \hat{v}, \hat{\eta})$. From that, one can derive the dispersion relation from the condition $\det A = 0$ with the 3×3 matrix originating from the algebraic equations $A(u, v, \eta)^T = 0$. The trivial solution $\omega = 0$ can be related to a steady (geostrophic) flow. For the non-trivial solution, the frequency ω is as in b) for $f = 0$ (classical gravity waves), and $\omega = f$ for low wave numbers $k, l \rightarrow 0$ (inertial oscillation, all fluid particles move in unison).

d) Kelvin wave. Assume a vertical wall at $x=0$ along the y -axis (an idealized coast) and $u=0$. Derive the solution for $v(x, y, t)$ and $\eta(x, y, t)$ using the equations (19,20)! Specify the x -dependence of the solutions using (18) and discuss the trapping distance from the coast!