Exercises for Dynamics I, 25. January 2007

1. Population Dynamics: Consider population dynamics with population $x > 0$ and reproduction (birth-death) r:

$$\frac{d}{dt}x = r(x)x \tag{1}$$

a) Solve the differential equation for $r = r_0 = const.$!
What happens for $t \to \infty$ when $r_0 > 0$ or $r_0 < 0$?
b) Solve the differential equation for $r = r_0(1 - x)$! (limited growth)
What happens for $t \to \infty$?
c) Consider the case $r = r_0(1 - x/K)$ with $K > 0$! Give a physical interpretation for K!

2. Difference equations: Consider the discretised form of 1. using the Euler scheme

$$\frac{d}{dt}x \approx \frac{x_{n+1} - x_n}{\Delta t} \quad . \tag{2}$$

a) Write down the iteration x_{n+1} as a function of x_n for the case 1a!
b) What is the solution of x_{n+1} as a function of x_0?
Consider the stability for the cases $r > 0$, $0 > \Delta t\, r > -1$, $-1 > \Delta t\, r > -2$, $-2 > \Delta t\, r$.
Do you have a graphical interpretation of the oscillation/decay?
c) Write down the iteration x_{n+1} as a function of x_n for the case 1b!

3. Bifurcation:
a) Consider the system 1b. Calculate the bifurcation with respect to parameter r!
Draw the bifurcation diagram!
b) as in a), but for

$$\frac{d}{dt}x = r_0 + x^2 \tag{3}$$

c) as in a), but for

$$\frac{d}{dt}x = x\sqrt{(r_0 + x)^2} \tag{4}$$

4. Stommel's Box model:
Consider the case when the temperatures and surface fluxes are fixed.
a) Assumption that the overturning rate Φ is constant and independent on salinity. What is the qualitative behaviour of the system (stability)? Would a freshwater perturbation in the northern box be reduced or increased?
b) Let us define the constant θ as the meridional temperature gradient, α and β the thermal and haline expansion coefficients. Assumption that the overturning rate $\Phi = c(\alpha\theta - \beta x)$ with x being the meridional salinity gradient. What is the qualitative behaviour of the system (stability)? Would a freshwater perturbation in the north be reduced or increased?