Coherence and Cross-spectra*

Coherence analysis, or cross-spectral analysis, may be used to identify variations which have similar spectral properties (high power in the same spectral frequency bands): if the variability of two distinct time series is interrelated in the spectral domain.

The coherence results are similar to the FFT results in a mathematical sense: real and imaginary coefficients. The application of a coherence analysis to two identical time series is the same as taking the Fourier transform, but the phase information is ``lost'' ($\Phi = 0$).

Can be thought of splitting the time series into two components: high (H) and low (L) frequencies: $x_t = x_t^{(H)} + x_t^{(L)}$**, and** $y_t = y_t^{(H)} + y_t^{(L)}$**. We may want to know whether the slow components of** x **and** y **vary together in some way.**

The cross-spectrum is defined from the covariance function C_{xy}**:**

$$\Gamma_{xy}(\omega) = \sum_{\tau=-\infty}^{\infty} C_{xy} \exp\{-2\pi i \tau \omega\}, \omega \in [-1/2, ..., 1/2].$$

Complex function: the power is $A_{xy}(\omega)^2 = \mathrm{Re}(\Gamma_{xy}(\omega))^2 + \mathrm{Im}(\Gamma_{xy}(\omega))^2$ **and the phase information is:**

$$\Phi_{xy}(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}(\Gamma_{xy}(\omega))}{\mathrm{Re}(\Gamma_{xy}(\omega))}\right).$$

A cross-spectrum for two similar processes, but with one shifted in time with respect to the other: $x(t)$ **and** $x(t+\tau)$**, gives the same power spectrum as for the same analysis applied to two identical time series,** $x(t)$**, but instead of a phase difference of zero, the phase is linear in frequency with a slope proportional to the phase shift:** $\Phi_{xy}(\omega) = 2\pi\tau\omega$**.**

The coherence spectrum is analogous to the conventional correlation coefficient and is defined

as:

$$\kappa_{xy}(\omega) = \frac{A_{xy}(\omega)^2}{\Gamma_{xx}(\omega)\Gamma_{yy}(\omega)}.$$

(Von Storch and Zwiers (1999) [], p. 234-241)
