

# Dynamical Climatology

Name:

email:

1) What are the periodicities in the Earth orbital parameters?

a) What is their physical meaning?

b) Are these orbital parameters independent?

2) What is the present day configuration?

3) Please draw the situation for 6 kyBP!

4) The covariance matrix  $C_x$  of a real process  $\vec{x}$  with mean  $\langle \vec{x} \rangle$  is defined as follows:

b) Is  $C_x = C_x^T$  ?

5) a) Let's once again let  $\vec{x}$  a measurement vector, and  $\vec{y} = M\vec{x}$  be a transformation to a new set of variables. Please write down  $C_y$  in terms of  $C_x$  and M !

b) What happens with  $C_y$  if the columns of  $M^T$  are taken to be the eigenvectors of M ?

c) Please describe in words and formula the singular value decomposition!

6) Please write down the definition of EOF and PC for the process  $\vec{x}$  !

b) What is the difference between T- and S-EOFs ?

7) While the preindustrial concentration of atmospheric  $^{12}CO_2$  amounted to about 280 ppm, the  $^{12}CO_2$  volume mixing ratio at the LGM was only about 200 ppm. Estimate the glacial value of atmospheric  $\Delta^{14}C$ , assuming that the cosmogenic production of  $^{14}C$  has not changed (which in fact is an oversimplification) and that  $\Delta^{14}C \approx \delta^{14}C$ .

8) Please solve the radioactive decay equation

$$\frac{d}{dt}x = -\lambda x + s$$

with a constant source term  $s$ .

b) Consider two different constituents,

$$x(0) = 1000, s = 10, \lambda = 10$$

$$y(0) = 100, s = 10, \lambda = 1$$

When is  $x = y$  ?

c) How can the equation be used to date substances?

9) Consider a reservoir with two separate sources  $Q_1$  and  $Q_2$ , and a single sink  $S$ . The magnitude of  $Q_1$  and  $S$  and their uncertainties have been estimated to be  $75 \pm 20$  and  $100 \pm 30$  (arbitrary units). Assuming that there is no direct way of estimating  $Q_2$ , how would you derive its magnitude and uncertainty range from budget considerations? tipp: assume equilibrium.

b) Regarding the uncertainty, which assumption must be made for the estimation of uncertainty?

c) What is the turnover time in this system?

10) Consider the non-linear case

$$\frac{d}{dt}M = Q - BM^2$$

with a constant source term  $Q$ , the removal rate is proportional to the square of the reservoir content. Assume  $M(0) = 0$ . Please solve and draw the solution!

11) Consider the logistic map:

$$\frac{d}{dt}M = AM - BM^2$$

with a const.  $A, B$ . This is an example for the growth of many biological systems. Calculate the solution for

$$M(0) = 0$$

$$M(0) = M_0 > 0$$

12) Classify the following models according to linear/non-linear, positive/negative feedback !

$$\frac{d}{dt}x = \lambda x + \textit{forcing}$$

with  $\lambda = \textit{const.} > 0$

with  $\lambda = \lambda_0 + \lambda_1 x, \lambda_0 < 0, \lambda_1 > 0$

13) What is the main idea behind the Milankovitch Theory of ice ages? Describe!

14) Describe the rectification process!