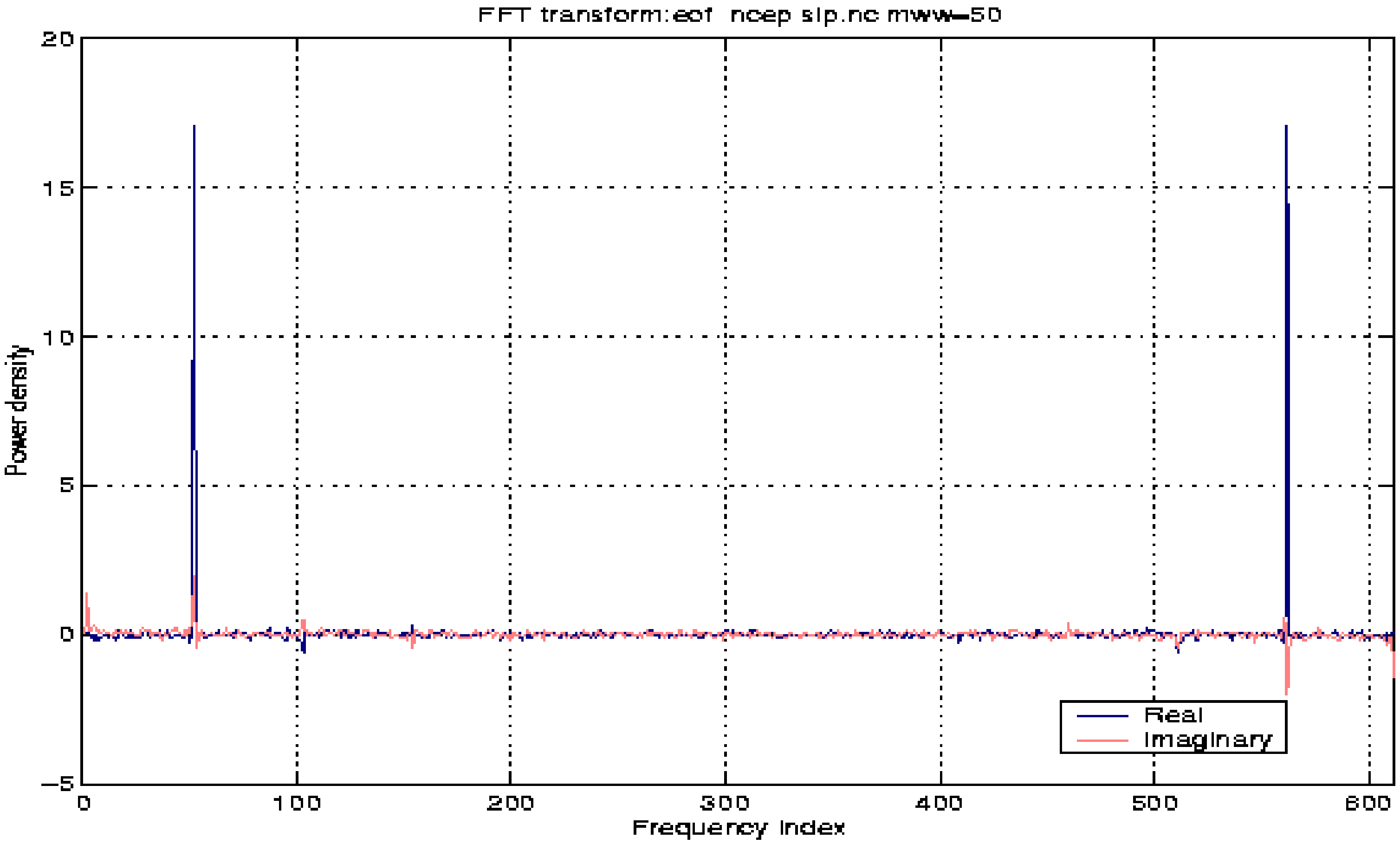
Fourier and Fast Fourier Transforms

Figure 10.1: *The FFT of the leading PC from NCEP SLP PC. [stats_uib_10_1.m]*



Any finite time series may be represented as a combination of sinusoidals of different harmonics:

$$f(x,t) = \sum_{\omega=0}^{\infty} \left[c_{x,\omega} \cos\left(\frac{2\pi\omega t}{L}\right) + s_{x,\omega} \sin\left(\frac{2\pi\omega t}{L}\right) \right]$$

The objective of spectral analysis is to find the variance (energy) associated with the various harmonics.

Fourier transforms (FT) are discussed in in general by Strang (1988) and the *numerical recipes* [,], whereas Wilks (1995) and Von Storch and Zwiers (1999) [,,] focus on the spectral analysis for climate studies.

The FT transforms data from time-domain (t) to frequency-domain (ω). A FT can be expressed

as $f(t) \rightarrow F(\omega)$, according to

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$$

The most common way to carry out an FT is by the *fast Fourier transform* (FFT) method, which is a fast numerical algorithm that transforms discrete data without solving the integral.

For any sampling interval, Δ, there is a critical frequency,

$$f_c = \frac{1}{2\Delta},$$

called the *Nyquist frequency*: the highest frequencies that can be represented by the spectral analysis correspond to two sample points per cycle. Frequencies higher than f_c will introduce distortions known as *aliasing* effects. It is therefore important that the spectral power densities in the periodograms tail off towards the high-frequency end.

Table 10.1: Properties of the FT

f(*t*) is real	$F(-\omega) = [F(\omega)]^*$
f(*t*) is imaginary	$F(-\omega) = -[F(\omega)]^*$
f(*t*) is even	$F(-\omega) = F(\omega)$
f(*t*) is odd	$F(-\omega) = -F(\omega)$